3.5/11 $\quad f(x) = \dfrac{4}{(3x^2-2x+1)^3}$

$$x=0 \xrightarrow{3x^2-2x+1} 1 \xrightarrow{\wedge 3} 1 \xrightarrow{\frac{4}{\square}} 4$$

$$f'(x) = \frac{\overset{f'}{(0)}\overset{g}{\left((3x^2-2x+1)^3\right)} - \overset{f}{(4)}\overset{g'}{\left(\frac{d}{dx}\left[(3x^2-2x+1)^3\right]\right)}}{\Big(\underset{g}{(3x^2-2x+1)^3}\Big)^2}$$

$\frac{d}{dx}(f(g(x))) = f'(g(x))\cdot g'(x)$

$$\frac{d}{dx}\left[(3x^2-2x+1)^3\right] = 3(3x^2-2x+1)^2\cdot(6x-2)$$

$f(x) = x^3 \qquad f'(x) = 3x^2$

$g(x) = (3x^2-2x+1) \qquad g'(x) = 6x-2$

$$f'(x) = \frac{\overset{f'}{(0)}\overset{g}{\left((3x^2-2x+1)^3\right)} - \overset{f}{(4)}\overset{g'}{\left(3(3x^2-2x+1)^2\cdot(6x-2)\right)}}{\Big(\underset{g}{(3x^2-2x+1)^3}\Big)^2}$$

$$\frac{-4\left(3(3x^2-2x+1)^2(6x-2)\right)}{(3x^2-2x+1)^6}$$

$$= \frac{(3x^2-2x+1)^2}{(3x^2-2x+1)^2}\cdot\frac{-12(6x-2)}{(3x^2-2x+1)^4}$$

3-5/11 redux

$$\frac{4}{(3x^2-2x+1)^3}$$

$$= 4\left(3x^2-2x+1\right)^{-3}$$

$$x \longrightarrow 3x^2-2x+1 \longrightarrow \wedge^{-3}$$

$$\frac{dy}{dx} = 4\left(-3\left(3x^2-2x+1\right)^{-4} \cdot (6x-2)\right)$$

$$4\left(-3\left(3x^2-2x+1\right)^{-3-1} \cdot \left(\frac{d}{dx}\left(3x^2-2x+1\right)\right)\right)$$

$$(2x+1)^{2003}$$

9) $y = \left(x^3 - \frac{7}{x}\right)^{-2}$

$\frac{d}{dx}\left(\frac{1}{x}\right) =$

$\frac{d}{dx}(x^{-1}) = -x^{-2}$

$= \frac{-1}{x^2}$

$x \longmapsto x^3 - \frac{7}{x} \longmapsto \wedge^{-2}$

$\frac{dy}{dx} = (-2)\left(x^3 - \frac{7}{x}\right)^{-3}\left(3x^2 + \frac{7}{x^2}\right)$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

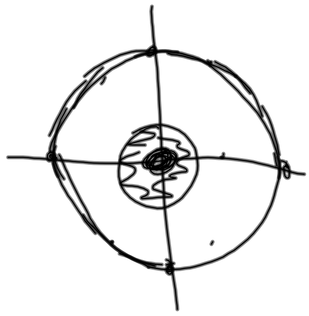
Anti-derivative of $3x^2$ is $(x^3 + C)$

$$\int 3x^2\,dx = x^3 + C$$

3.6) Implicit Differentiation

$y = f(x)$ $\quad$ $x^2 + y^2 = 36$

x	y^2	y
0	36	$-6, +6$
6	0	0
-6	0	0
3	27	$\pm 3\sqrt{3}$



$$\frac{d}{dx}\left(x^2 + y^2 = 36\right)$$

$$2x + 2(y)\frac{dy}{dx} = 0$$

$$\frac{d}{dx}\left(y^2\right) \qquad \frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\frac{d}{dx}\left[\left(g(x)\right)^2\right] \qquad \frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$$x \rightarrow g(x) \mapsto {}^2$$

$xy = 4 \longrightarrow y = \frac{4}{x}$

$\Downarrow$ implicitly

$\frac{dy}{dx} = -\frac{4}{x^2}$

$\frac{d}{dx}(x \cdot y)$

$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = \frac{-y}{x}$

$= \frac{-\left(\frac{4}{x}\right)}{x} = \frac{-4}{x^2}$

13) $\frac{d}{dx}\left(\left(4+(3x)^{1/2}\right)^{1/2}\right)$

$x \longmapsto 3x \longrightarrow \cap^{\frac{1}{2}}+4 \longrightarrow \cap^{1/2}$

$f(x)=x^{1/2} \longrightarrow \underline{f'(x)}=\frac{1}{2}x^{-1/2}$

$g(x)=4+(3x)^{1/2} \qquad g'(x)=\frac{d}{dx}\left((3x)^{1/2}\right)$

$x \to 3x \to \cap^{\frac{1}{2}}$

$h(x)=x^{1/2} \qquad h'(x)=\frac{1}{2}x^{-1/2}$

$i(x)=3x \qquad i'(x)=3$

$g(x)$

$\frac{1}{2}\left(4+\sqrt{3x}\right)^{-1/2}\cdot\left(3x\right)^{1/2}\cdot 3$

Note that $\sqrt{3x}=\sqrt{3}\sqrt{x}$

and $\sqrt{3}$ is a number.

$\left(4+\sqrt{3}x^{1/2}\right)^{1/2}$

$x \to 4+\sqrt{3}x^{1/2} \to (\quad)^{1/2}$

$x=4$

$4 \to 2\cdot\sqrt{3}+4 \to \cap^{\frac{1}{2}}$

3.5/9

$$y = \left(x^3 - \frac{7}{x}\right)^{-2}$$

$$x \longmapsto \left(x^3 - \frac{7}{x}\right) \longrightarrow \square^{-2}$$

composition of fns

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x)) \cdot g'(x)$$

$$f(x) = x^{-2} \qquad f'(x) = -2x^{-3}$$

$$g(x) = x^3 - \frac{7}{x} \qquad g'(x) = 3x^2 + \frac{7}{x^2}$$

$$\frac{7}{x} = 7(x^{-1})$$

$$\frac{d}{dx} x^{-1} = (-1)(x^{-2})$$

$$\frac{dy}{dx} = -2\left(x^3 - \frac{7}{x}\right)^{-3}\left(3x^2 + \frac{7}{x^2}\right)$$

3.5/12

$f(x) = \sqrt{x^3 - 2x + 5}$

$= (x^3 - 2x + 5)^{1/2}$

$f'(x) = \frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}}(3x^2 - 2)$

$f'(x) = \frac{3x^2 - 2}{2\sqrt{x^3 - 2x + 5}}$

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$g(x) = x^3 - 2x + 5$

$g'(x) = 3x^2 - 2$

$y = \sin\left((5-2x)^4+7\right)$

$f(x) = \sin x$ $\quad g(x) = x^4+7$ $\quad h(x) = 5-2x$

$f'(x) = \cos x$ $\quad g'(x) = 4x^3$ $\quad h'(x) = -2$

$4(5-2x)^3 \cdot -2$ $\qquad 4(5-2x)^3 \cdot (-2)$

$\left\{f(g(x))\right\}' = f'(g(x)) \cdot g'(x)$

$\cos\left((5-2x)^4+7\right) \cdot \frac{d}{dx}\left[\quad\right]$

$\cos\left((5-2x)^4+7\right) \cdot \left(4(5-2x)^3 \cdot (-2)\right)$

$$y = \frac{\sin\left(\frac{x^3+3}{\cos(x^2)}\right)}{\cos\left(\sin(x^2+7)\cdot\sin(3-\cos(x^2))\right)}$$

$$\frac{\left[\frac{d}{dx}\left[\sin\left(\frac{x^3+3}{\cos(x^2)}\right)\right]\right]\left[\cos\left(\sin(x^2+7)\cdot\sin(3-\cos(x^2))\right)\right]-\left[\sin\left(\frac{x^3+3}{\cos(x^2)}\right)\right]\left(\frac{d}{dx}\left(\cos\left(\sin(x^2+7)\cdot\sin(3-\cos(x^2))\right)\right)\right)}{\left[\cos\left(\sin(x^2+7)\cdot\sin(3-\cos(x^2))\right)\right]^2}$$

$$\frac{d}{dx}\left(\sin\left(\frac{x^3+3}{\cos(x^2)}\right)\right) = \cos\left(\frac{x^3+3}{\cos(x^2)}\right)\left(\frac{d}{dx}\left(\frac{x^3+3}{\cos(x^2)}\right)\right)$$

$$\frac{d}{dx}\left(\frac{x^3+3}{\cos(x^2)}\right) = \frac{(3x^2)(\cos(x^2)) - (x^3+3)\left(\frac{d}{dx}(\cos(x^2))\right)}{(\cos(x^2))^2}$$

$$\frac{d}{dx}\left(\cos(x^2)\right) = -\sin(x^2)\cdot(2x)$$

3.6) Implicit Differentiation

$y = f(x)$ $\qquad$ $x^2 + y^2 = 36$

$x + xy = 14$ $\qquad$ $xy = 4$

[remember.... $y = f(x)$]

$$\frac{d}{dx}(x + xy = 14)$$

$$1 + \left[(1)(y) + (x)\left(\frac{dy}{dx}\right)\right] = 0$$

$$1 + y + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = -y - 1$$

$$\frac{dy}{dx} = \frac{-y-1}{x}$$

Antiderivatives

If $\frac{d}{dx}(x^3) = 3x^2$

then the

antiderivative of $3x^2$ is $x^3 + C$.

$$\int 3x^2 \, dx = x^3 + C$$

~~Note~~ $4+\sqrt{3x} = 4+\sqrt{3}\sqrt{x}$

(3) $\dfrac{d}{dx}\left(\left(4+(3x)^{1/2}\right)^{1/2} \right)$

$= \dfrac{1}{2}\left(4+(3x)^{1/2}\right)^{-\frac{1}{2}} \cdot \dfrac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3$

$= \dfrac{3}{4\sqrt{4+\sqrt{3x}}\cdot\sqrt{3x}}$

$\dfrac{d}{dx}\left(\left(4+\sqrt{3}x^{1/2}\right)^{1/2} \right)$

$= \dfrac{1}{2}\left(4+\sqrt{3}x^{1/2}\right)^{-\frac{1}{2}}\left(\sqrt{3}\left(\tfrac{1}{2}x^{-1/2}\right)\right)$

$\left(4+(3x)^{1/2}\right)^{1/2}$

$f(x) = (\quad)^{1/2}$

$g(x) = 4+(\quad)^{1/2}$

$h(x) = 3(\quad)$

$\sin^3 x =$

$$(\sin x)^3$$